



Optimisation of Distributed Generation using Simplex Algorithm

Murdoch University ENG460:
Engineering Thesis Project 2014

Kimberley Francis

Contents

Table of Tables	3
Table of Figures	4
Acknowledgement	5
1 Executive summary	6
2 Background to DG	7
2.1 What is DG	7
2.2 Benefits of DG	7
2.3 Problems associated with DG	8
2.3.1 Voltage fluctuation and regulation	8
2.3.2 Power Fluctuation	9
2.3.3 Voltage frequency harmonics	9
2.4 “Green” DG Technologies	9
2.4.1 CHP	9
2.4.2 Wind Power Plants	10
2.4.3 Solar photovoltaic	10
3 DG Case Studies	11
3.1 Carnarvon PV Generation	11
3.2 Kalbarri Wind Farm	11
4 Introduction to the Model	13
4.1 Operational Cost	13
4.2 Voltage stability	14
4.2.1 Line Voltage stability	14
4.2.2 Reverse power flows	14
4.3 Line losses	15
4.3.1 DG impact on line losses.	15
5 Model Part A - linear power/voltage relationships at the buses.....	16
5.1 Linear programming.....	16
5.2 The power system network	16
5.2.1 System Parameters	17
5.3 Ad-hoc placement of generators in the network	19
5.4 Power flow simulation	20
5.4.1 Power injected into Bus 1	20

5.4.2	Power injected into bus 2	21
5.4.3	Power injected into bus 3	22
5.4.4	Voltage Dependencies	23
5.5	Linear equation and constraints	24
5.5.1	Objective function.....	24
5.5.2	Constraints	24
5.5.3	Voltage and Power characteristics.....	25
5.6	Optimisation and Results	26
5.7	Results for Part A of the Model.....	26
6	Model Part B – line losses	28
6.1	Simplex programming	28
6.2	System model.....	28
6.2.1	Power injected into Bus 1	29
6.2.2	Power injected into Bus 2	30
6.2.3	Power injected into Bus 3	31
6.3	Quadratic equation and constraints	33
6.3.1	Objective function.....	33
6.3.2	Constraints	34
6.3.3	Inequalities.....	35
6.3.4	Optimisation.....	35
6.4	Results for Model Part B	36
7	Future work.....	38
8	Conclusion.....	39
	Appendix A: Matlab code.....	40
	Appendix B: Scilab code	41
9	References	42

Table of Tables

Table 1 Effects of high levels of PV penetration on the Carnarvon grid (Lewis 2012).....	11
Table 2 System bases	18
Table 3 Cable Resistance and Reactance factors	18
Table 4 Network Resistance and Reactance	18
Table 5 Transformer Resistance and Reactance	18
Table 6 Effect of on Voltages of Power injected at Bus 1.....	20
Table 7 Effect on Voltages of Power injected at Bus 2.....	21
Table 8 Effect on Voltages of Power injected at Bus 3.....	22
Table 9 Voltage interdependency	24
Table 10 Generated Power – Part 1.....	26
Table 11 Generated power and associated voltages – Part A	26
Table 12 Power and line losses for power injections at bus 1	30
Table 13 Power and line losses for power injections at bus 2	31
Table 14 Power and line losses for power injections at bus 3	32
Table 15 line loss coefficients	33
Table 16 Generated Power – Part B.....	36
Table 17 Generated power and associated voltages – Part B	36
Table 18 Part A and Part B results compared	37

Table of Figures

Figure 1 power system network	17
Figure 2 Line diagram with generators operating at 10-15 MW capacity	19
<i>Figure 3 Voltage dependencies of Power injection at bus 1</i>	<i>21</i>
<i>Figure 4 Voltage dependencies of Power injection at bus 2</i>	<i>22</i>
<i>Figure 5 Voltage dependencies of Power injection at bus 3</i>	<i>23</i>
Figure 6 Line diagram with loads at each bus set to zero.....	29
Figure 7 Graph of power and line losses for power injections at bus 1	30
Figure 8 Graph of power and line losses for power injections at bus 2	31
Figure 9 Graph of power and line losses for power injections at bus 3	32
Figure 10 Overall loss characteristics.....	33

Acknowledgement

I'd like to thank Dr Gregory Crebbin for all his support and guidance throughout this project, thank you for always making time to answer my many queries. I'd also like to acknowledge the other teaching staff in the school of Engineering and Information Technology at Murdoch University for their contribution to my learning through the course of my undergraduate degree.

I'd like to thank my family, without whose encouragement and constant gentle nagging this project, and degree, would not have been possible, thank you Paul, Lorna, Jen, Matthew, Pat, Lorraine and Lucy. I love you all and appreciate you more than you could ever know.

Lastly a special thank you to Paul for always being willing to proof read documents at the very last minute, you're a wonderful father and have given me the self-belief I need to succeed.

Kimberley Francis

July 2014

1 Executive summary

The traditional centralised power grid was based upon the concept that central power sources would feed power through the transmission system and then the distribution system where the power would be utilised by the connected loads (ITP 2010).

The concept of the traditional power grid is changing with the increasing connection of distributed generation (DG) or embedded generation sources (Jenkins, et al. 2010). Grid controls and operations need to be redesigned for DG sources which introduce conditions, such as voltage rise and reverse power flow, for which a grid may not have been designed (Jenkins, et al. 2010). Case studies of DG implementation in Western Australia provide practical experience of the results of connecting DG sources to existing grids (Lewis 2012).

This thesis describes a practical method for determining the optimum generation of power in a network and minimising line losses using DG sources while keeping bus voltages within required limits. While power flow simulations can be used to optimise networks, their use can be time consuming, particularly if the system is not simple. This paper uses minimal power flow simulations and provides mathematical models of the network conditions.

The method proposed in this paper aims to reduce the time taken to produce a power flow optimisation problem in three steps:

1. performing power flow simulations to develop a linear approximation of a non-linear model of power and voltage relationships with injected power
2. performing further power flow simulations to develop a non-linear model of power and line loss relationships
3. then using both the linear power/ voltage and non-linear power / line loss relationships in a simplex algorithm to determine the optimum solution for generator location and capacity with minimum power loss while maintaining voltage stability

Future work suggested includes extending the model to include thermal limits, transformer limits and short circuit ratings.

2 Background

The idea of the distribution system as passive is no longer valid with the increasing amounts of DG included currently and projected for the near future in power systems (Jenkins, et al. 2010). The traditional centralised power grid was based upon the concept that central power sources would feed power through the transmission system and the distribution system where the power would be utilised by the connected loads (ITP 2010).

Grid protection, control power flows and earthing were generally not designed to accept generation from the distribution system (ITP 2010). Due to the lower operational cost of renewable energy sources and growing awareness of climate change, greater use is being made of renewable energy sources (Milligan, et al. 2012).

At low levels of DG penetration (i.e. 0-10% of the connected load) it is considered that the implementation of DG within existing distribution systems is unlikely to lead to significant problems (ITP 2010). Higher levels of DG penetration can cause serious problem such as voltage rise and reverse power flow within the grid, in particular within the distribution system which has fewer controls than the transmission system (Jenkins, et al. 2010).

This thesis suggests a method for determining the optimum generation of power in a network and minimising line losses using DG sources while keeping bus voltages within required limits. These results can be achieved using power flow simulations but this paper uses minimal power flow simulations and provides mathematical models of the network conditions.

2.1 What is DG

In contrast to large centralised power generation, DG utilises dispersed power generating sources to supply loads, with the power supply typically being close to the demand. DG can provide for local demand only, or can be integrated into a supply grid and provide power to a wider system.

DG can be provided from renewable sources (solar, wind), fossil fuels or waste heat (Jenkins, et al. 2010). The equipment can provide power ranging from less than a kilowatt up to tens of megawatts. DG and renewable energy sources can reduce dependency upon fossil fuels and can reduce emissions of greenhouse gases.

Use of DG is increasing world-wide but the future of DG is subject to regulations and government support (Lai and Chan 2007). Distributed generation regulations generally originate from the transmission and distribution companies responsible for the reliable supply of power and aim at limiting the power sourced from intermittent renewable sources to reduce possible negative effects on the network (Pisica, Postolache and Edvall, Planning of Distributed Generation via Nonlinear Optimization and Genetic Algorithms 2010).

2.2 Benefits of DG

The main benefits of DG are:

- Reduction in power distribution costs - DG can assist to reduce costs of power distribution by reducing the requirement for long high voltage transmission lines, sub stations and transformers (Lai and Chan 2007).
- Reduction in line losses – in a centralised grid system, power is often transmitted over long distances, resulting in power losses in the network lines. DG can reduce line losses as power is generated close to the demand (Pisica, Postolache and Edvall 2010).
- ‘Cleaner’ electricity production - renewable DG energy sources include solar photovoltaic (PV) arrays, wind turbines, gas and biomass generators etc. (Keane and O'Malley 2005). There are world-wide efforts to reduce greenhouse gas emissions and utilising more renewable energy sources can aid in the effort to reduce such emissions.

2.3 Problems associated with DG

Problems in the distribution system that can be associated with DG include:

- Voltage fluctuation and regulation
- Power fluctuation
- Voltage frequency harmonics (ITP 2010).

2.3.1 Voltage fluctuation and regulation

2.3.1.1 Voltage levels and reverse power flow

The centralised electricity grid was designed for power flows in one direction from a central source to distributed loads through substations, transmission lines, transformers and distribution networks. The network allows for voltages that are often 5-10% higher than the required voltage of the customer, to allow for voltage drops along the lines (Jenkins, et al. 2010). Large levels of DG in one area can increase the expected voltage levels (referred to as voltage rise) and possibly reverse the direction of the power flow (ITP 2010).

Voltage rise can lead to problems if voltages exceed the limits required to ensure correct operation of customers’ appliances. Reverse power flow can create difficulties where the grid is not designed to provide adequate protection and control for such conditions.

2.3.1.2 Voltage fluctuations

Due to anti-islanding safety regulations, inverters connecting DG sources to the network are designed to disconnect from the grid if the grid voltage or frequency exceeds certain boundaries. If the grid voltage fluctuates for a short time then all the inverters will suddenly disconnect leaving the centralised generators to suddenly provide extra power (ITP 2010).

2.3.1.3 Voltage imbalance

Voltage imbalance may also occur if the DG is not connected equally over all three phases of the system and can cause problems with transformers and power electronic devices such as prematurely aging the equipment (Lai and Chan 2007).

2.3.2 Power Fluctuation

Power from renewable energy sources can vary throughout the day, changing by the hour and even minute. Power output from solar panels can be significantly reduced when the panels are shaded by clouds. Output from wind turbines can suddenly reduce from full power to nothing when the wind speed exceeds the rated range and the turbine safety systems stop the rotors from turning. These fluctuation in power can cause flicker, so-called as it refers to the flickering of luminescent lights caused by an unstable voltage (Santjer 2005).

It can be difficult to integrate renewable sources into systems with a large base load power plants, particularly nuclear or coal power plants as they cannot rapidly alter their output. Gas turbines, which are more flexible are a better option in combination with DG sources (Jenkins, et al. 2010).

2.3.3 Voltage frequency harmonics

Ideally the voltage in a network has a perfect sinusoidal form but switching gear and inverter connections can introduce harmonics that will alter the sinusoidal output. (Lai and Chan 2007)

2.4 “Green” DG Technologies

DG can use a wide range of technologies from proven approaches such as Combined Heat and Power (CHP, also referred to as cogeneration) and reciprocating internal combustion engines to more recent developments using wind power, solar power and fuel cells (Jenkins, et al. 2010).

Each technology and energy source has its own distinctive features which affect the interaction of DG with the established power generation and transmission systems. Network problems can be increased or eased with the establishment of DG, and regulations and contracts between the owners and operators of the DG, transmission and generation facilities are very important in determining the success of DG (Jenkins, et al. 2010). In addition fuel costs and availabilities in different regions also affect the feasibility of DG in those locations.

Some of the more commonly used DG technologies which utilise sustainable fuel sources are described below.

2.4.1 CHP

CHP is the production of electrical power together with useful heat and is an important DG technology (Jenkins, et al. 2010). Typically the power generated is used locally with any excess or deficiency being supplied to or from the external network. The heat is used locally or provided to the surrounding area for heating. The use of district CHP heating is common in Denmark, Sweden and Finland (Lai and Chan 2007). Industrial CHP installations can use up to 35% less primary energy than central power stations and also achieve a 10 to 30% reduction in CO₂ emissions (Jenkins, et al. 2010).

Jenkins et al 2010 refer to dispersed CHP plants in Denmark that could be run at a time of high electricity demand and included heat stores that could retain heat for up to 10 hours thus allowing

the CHP units to maximise both electrical and heat outputs and minimise plant operations when the electrical and heat demands did not occur at the same time.

CHP fuels include biomass, including municipal and other waste and landfill gases, and commercial hydrocarbons.

2.4.2 Wind Power Plants

Wind Power Plants function by the kinetic energy of the wind turning the rotor blades of a turbine (Milligan, et al. 2012). The power of the turbine is proportional to the cube of the wind speed so a high mean annual wind speed is important in determining the location of wind power plants. A high mean annual wind speed alone does not guarantee continual power and Jenkins et al. 2010 show results demonstrating that a wind turbine operating at a UK lowland site might not operate due to low wind speeds for up to 25% of the time and output during operation will vary depending upon wind speed.

The turbines used in wind farms can either be fixed speed (generally using induction generators) or variable speed (either synchronous or induction generators). Fixed speed induction generators are affected by the cyclic torque resulting from tower shadow, wind shear and turbulence as the blades pass the tower although the combined effect of a wind farm is to average out the variations (Holtinen and Hirvonen 2005). In addition attention needs to be paid to start up and to transient output power, which has been found to be up to twice the nominal output power in some instances (Jenkins, et al. 2010).

Large wind farms have been installed and more are planned in Europe and also in China and North America (Global Wind Energy Council 2013). Availability of land can be a significant constraint for wind farms. Obtaining planning permission is also a major factor, even for offshore wind farms. Other key considerations include maintenance and connection to the network.

2.4.3 Solar photovoltaic

Solar photovoltaic (PV) generation is a well-accepted power generation technology that ranges in scale from household installations typically of about 2-3 kW to large arrays providing tens or even hundreds of MW (Jenkins, et al. 2010). Output is maximised on a clear day where sunlight strikes the cell or array surface directly. However even in the absence of direct sunlight, some output is obtained from diffused sunlight.

Many developed countries permit households to link PV arrays to power grids provided specified standards are followed. The popularity of household PV arrays in Australia has raised concerns from transmission and distribution providers such as Western Power and Horizon as large amounts of rooftop PV in one area can have an effect upon the electricity grid (Lewis 2012).

A study of high rooftop solar in a small town that highlights some associated issues will be outlined in the next section.

3 DG Case Studies

3.1 Carnarvon PV Generation

Carnarvon is an isolated community in WA's North West. Carnarvon's network is comprised of a diesel/gas powered grid and includes a large amount of distributed generation in the form of household PV systems. This dispersed PV generation makes a substantial impact as it is teamed with strong solar irradiation in the area (Lewis 2012). The PV penetration in Carnarvon is said to make up 13% of the load at peak solar irradiation times, which is a reasonably high level of penetration. A case study was complete in 2012 for the Australian Solar Institute, which looked at the effects of high levels of PV penetration in an isolated supply system (Lewis 2012). Table 1 below summarises some of the effects that high levels of PV penetration have had on this system.

Power fluctuation due to inverter anti Islanding protection	There has been a recorded case where large numbers of PV inverters disconnected due to a system frequency problem. This meant the central generator had to rapidly meet this load.
Voltage rise	There have been two cases where voltage rise has been noted. The tap changing transformer was set too high causing the voltages to rise above acceptable limits.
Reverse power flow	Backfeeding occurs through the distribution transformers but currently does not appear to have any effects on transformer operation.
Power fluctuations due to renewable resources	It was thought that having such high levels of PV penetration could cause problems with power fluctuations if there was a sudden drop in irradiation levels. So far this problem has not been observed.
System Harmonics	The high levels of PV in Carnarvon do not seem to be introducing any unnecessary currents into the system.
Reduction in fuel	Due to the high solar irradiation the centralised generator is being used less and this has resulted in a fuel saving.

Table 1 Effects of high levels of PV penetration on the Carnarvon grid (Lewis 2012).

1.1MW of PV capacity is currently installed and Horizon Power has set the maximum limit of PV penetration at 1.15MW (Lewis 2012). The limit on the level of PV generation is to ensure that the diesel and gas generators are still able to run at a reasonably high load so as not to be inefficient.

3.2 Kalbarri Wind Farm

A wind farm comprising two 850kW synchronous generators was commissioned at Kalbarri in 2007 (Synergy 2014). The wind farm also includes voltage control equipment in the form of an IGBT statcom for injection or absorption of reactive power, and undervoltage ride-through capabilities, should the 33kV line that it is connected to suffer from a voltage dip (Verve Energy 2006). In addition

there are two voltage regulating transformers along the line which adjust their tap values every minute so as to maintain the voltage within the $\pm 10\%$ (Verve Energy 2006).

Despite the inclusion of voltage control equipment, the wind farm can still affect the grid with Line Drop Compensation (LDC) and power quality and voltage change. Western Power uses LDC to improve the voltage profile on rural 22kV and 33kV distribution lines (Western Power 2008). LDC refers to the fact that substation transformers tap up their voltage levels as their loading increases to partially compensate for the voltage drop along the heavily loaded rural feeders and ensure that the voltage at the end of the feeders isn't unacceptably low. DG connections at the distribution level can reduce the zone substation loading and have negative effects on the line drop compensation unless the generator is located towards the end of the feeder (Western Power 2008). The Kalbarri Wind Farm is situated just 20km from the end of a long feeder and generally supports the feeder near the end of the line so the reduction that it causes in the line drop compensation is acceptable.

For the Kalbarri wind farm careful sizing and location planning has been undertaken to minimise or control any negative effects on the grid.

4 Introduction to the Model

This thesis describes a practical method of determining the maximum use of DG in a particular network, while minimising line losses and keeping bus voltages within limits

One way to understand the relationships between power loss and voltage is through the use of an efficient software power flow simulation. The optimisation problem can be performed through a trial and error approach in the simulation program with continual power flow simulations being run and results compared other until the optimum is found. This can be a lengthy process, especially if the system is quite complex (Pisica, Postolache and Edvall, Handbook of Power Systems 1:Planning of Distributed Generation via Nonlinear Optimization and Genetic Algorithms 2010).

The method proposed in this paper aims to reduce the time taken to produce a power flow optimisation problem in three steps:

4. performing power flow simulations to develop a linear approximation of a non-linear model of power and voltage relationships with injected power
5. performing further power flow simulations to develop a non-linear model of power and line loss relationships
6. then using both the linear power/ voltage and non-linear power / line loss relationships in a simplex algorithm to determine the optimum solution for generator location and capacity with minimum power loss while maintaining voltage stability

The model was performed and documented in two parts:

- Part A determines the linear power/voltage relationships at the buses (step 1)
- Part B uses the results from Part A and includes non-linear line losses (steps 2 and 3).

This model and methods are based on two research papers by Keane and O'Malley who used a linear program with numerous constraints, such as thermal limits and equipment short circuit ratings. This thesis uses a simplex algorithm with fewer constraints to achieve a similar objective of optimal allocation of distributed generation in order to reduce line losses (Keane and O'Malley 2005) (Keane and O'Malley 2006).

The operational factors considered in the model are described below.

4.1 Operational Cost

Once renewable energy sources have been installed their running costs are very low compared with other power plants as unlike other sources, there is no expensive fuel cost. For lowest cost therefore, the less expensive renewable generated power would ideally be used to supply the load first and other forms would be used to make up any shortfall.

Generation of power from renewable sources is subject to environmental conditions at the time of power generation and is less predictable than traditional central power generation in delivering a given quantity of power at any particular time.

Of the traditional power sources coal is a cheaper source than gas. However, power generation from coal cannot be quickly increased or decreased to cover any large fluctuation from renewable sources whereas solar generators and wind turbines can be limited much more easily; PV receivers can be tilted away from the maximum power point and wind turbines can be turned out of the wind (Jenkins, et al. 2010). It therefore becomes more practical to rely on coal-fired stations (for example) for the base load and to switch in and out any DG depending upon the extent of the variable load to be met, even though the power generated from the DG sources is less expensive.

If there is a correlation between the load profile and the DG power availability, e.g. the peak demand meets the peak generating capacity, then the DG power is of higher value as it is produced at a time when consumption is high.

4.2 Voltage stability

4.2.1 Line Voltage stability

The impedance of the lines in a distribution system and the load current cause a voltage drop along the distribution line (Northcote-Green and Wilson 2007). As there are normally variations in the load throughout the day, with peaks typically occurring in the evening, the load current varies with the load demand. When the load current increases so does the voltage drop along the distribution feeder (Northcote-Green and Wilson 2007). This natural voltage drop can cause problems as voltage delivered to customers is expected to be maintained at certain levels.

Voltage levels within the network must be kept within a narrow band of acceptable limits in order to perform its main function of transporting power from generators to loads (Slootweg, et al. 2005) and meet the voltage limits of connected. Traditionally the voltage level within transmission grids was controlled by large power plants while the voltages on the distribution system used voltage control devices such as tap changing transformers and capacitor banks. A tap changing transformer has a variable turns ratio so that the output voltage can be adjusted as desired, with either manual or automatic control. Where a tap changing transformer is positioned at the start of a distribution feeder (the connection point of the distribution grid to a higher level) the transformer will control the voltage of all the buses that feed from that distribution line and so can raise or lower all the voltages attached to that feeder (Slootweg, et al. 2005). Capacitor or reactor banks can either consume or supply reactive power and so change the voltage magnitude mainly at the bus to which they are connected (Slootweg, et al. 2005).

The connection of DG sources to the distribution system affects the power flow and node voltages (Slootweg, et al. 2005). Tap changing transformers are commonly used to regulate network voltages. However if these transformers have a set value then manual adjustment may be needed to maintain the voltage within the specified limits (Western Power 2008).

4.2.2 Reverse power flows

As mentioned above, the addition of large amounts of DG power to the distribution system can have a negative impact on load voltages and power flows (Slootweg, et al. 2005). The addition of DG at some point along the feeder will raise the voltage at this point. Should higher voltage exist at the DG

than at the start of the distribution feeder, reverse power flow may result. Protection equipment in the system is not always designed to cater for such reverse power flows therefore power utilities need to set up appropriate system protection in areas where reverse power flow could occur.

4.3 Line losses

There are two types of technical losses in power systems, permanent or variable. Permanent losses do not vary in relation to the current (and thus the load) and are caused mainly by the excitation of the distribution level transformers (Eckles 2007). Anywhere between 25 to 33% of technical losses on the distribution system are permanent losses (Parmar 2013). Variable losses are proportional to the square of the current (Parmar 2013). As the load demand increases so does the current needed and this in turn means an increase in the variable losses.

4.3.1 DG impact on line losses.

Wind and solar farms can have a significant impact on line losses in both transmission and distribution systems. The distance of the generators from the load can either increase or decrease the variable line losses (Holtinen and Hirvonen 2005). In WA the wind farms located in Kalbarri and Broome are located some distance from the load centre so the power they produce is transported which would lead to larger system losses. The Albany and Mt Barker wind farms are located nearer to their respective load centres so the power produced by those wind farms is used near its point of production which will mean the system losses are reduced.

5 Model Part A - linear power/voltage relationships at the buses

As described earlier (Section 4), the method proposed in this paper aims to produce a practical power flow optimisation problem in two steps:

- performing power flow simulations to develop a linear model of power and voltage relationships with injected power
- performing further power flow simulations to develop a non-linear model of power and line loss relationships and then uses both the linear power/ voltage and non-linear power / line loss relationships in a simplex algorithm to determine the optimum solution for generator location and capacity with minimum power loss while maintaining voltage stability

Part A determines the linear power/voltage relationships at the buses.

Development of the model commenced by describing the network to be used, examining the results arising from the ad-hoc placement of DG, and then subjecting the network to restrictions designed for operational stability and efficiency.

The system was modelled in Power World in order to perform load flow analysis and from these results assess the impact of increasing levels of distributed generation on bus voltages (Power World 2012).

5.1 Linear programming

The linear approximations were then used in a linear program to obtain the placement and capacity of the distributed generation.

Ferguson describes a linear programming problem (Ferguson n.d.) as the problem of maximizing or minimizing a linear function, known as the objective function, subject to linear constraints. The constraints may be equalities or inequalities.

5.2 The power system network

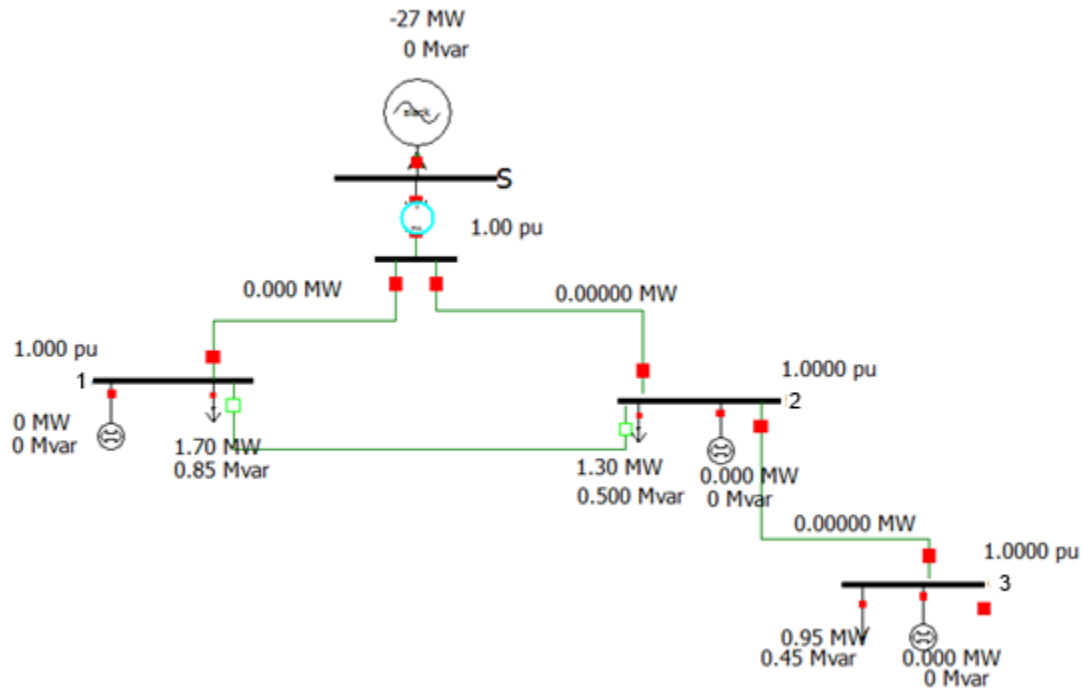


Figure 1 power system network

This power network is a simplified section of the distribution network used by Keane and O'Malley in their work on Optimal Allocation of Embedded Generation on distribution networks (Keane and O'Malley 2005).

The power network system in Figure 1 above has been modelled to represent a network to which DG can be added. The network is connected to the transmission grid - shown as a single generator in series with a transformer. Bus S, which connects the transmission grid to the network has been chosen as the system slack bus, which means that the active and reactive power provided by this bus will vary, but bus voltage magnitude and angle are set. Buses 1, 2 and 3 are load buses with set values for active and reactive power demands. The line joining bus 1 to 2 is normally open. Each bus has an active and reactive component to its load. The parameters for the distribution lines and transformers are shown in section "System Parameters" below.

The possible locations for the distributed generators to be placed are at bus 1, 2 or 3. The decision for the placement and capacity of the generation will depend on the impact of the generator on voltage at the nearest bus and also its impact on those surrounding it.

The case that is considered for this system is when the loads are at a minimum and the distributed generation is at its peak, as this is when there is greatest variation in voltages for the system.

5.2.1 System Parameters

The parameters of the system are shown in Tables 2-5 below

Per Unit bases	Values
Vbase	22kV
Sbase	20MVA
Zbase	24.2Ω

Table 2 System bases

Cable	ohms/km
Resistance	1.056
Reactance	0.328

Table 3 Cable Resistance and Reactance factors

Line	Length (km)	Resistance (ohms/km)	Reactance (ohms/km)	Resistance (pu)	Reactance (pu)
Grid 1	2	2.112	0.656	0.087273	0.027107
Grid 2	1	1.056	0.328	0.043636	0.013554
2 to 3	20	21.12	6.56	0.872727	0.271074
1 to 2	4	4.224	1.312	0.174545	0.054215

Table 4 Network Resistance and Reactance

Transformer	
Resistance (p.u)	0.00994
Reactance (p.u)	0.2088

Table 5 Transformer Resistance and Reactance

5.3 Ad-hoc placement of generators in the network

The ad-hoc placement of generators can increase system losses and cause reverse power flow in areas of the grid that have not been designed for such an occurrence. Having each generator operate at a capacity of (say) 10 - 15MW can adversely affect the system as the voltage levels at the buses will have increased past the optimum and then decreased again.

The effect of operating generators at capacities of 10-15 MW are shown in Figure 2 below.

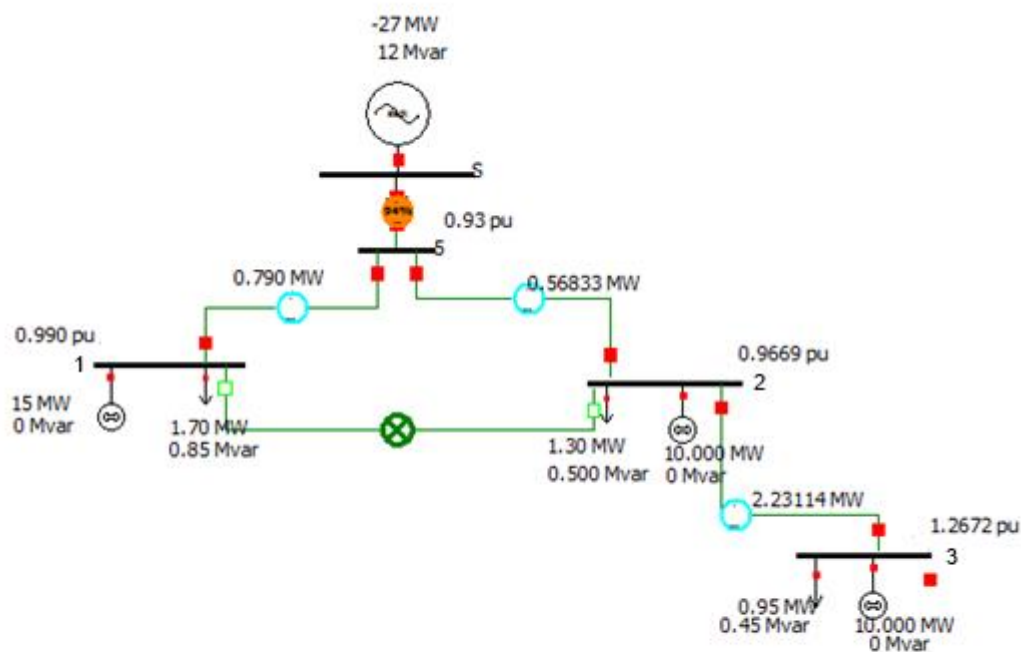


Figure 2 Line diagram with generators operating at 10-15 MW capacity

In order to demonstrate that voltage rise can occur with ad-hoc placement distributed generation was added to buses A, B and C. As can be seen in the single line diagram of a distribution system a 10MW generator added to bus 3 would cause the voltage to rise above the specified limits of $\pm 10\%$ as outlined by transmission and distribution companies (Western Power 2008). In this case the voltage in this at bus 3 is 27.878 kV, which is outside the band specified for correct operation of appliances and equipment for the customer.

The ad-hoc placement of generators will now be compared with a calculated method of optimising the distributed generation subject to generator limits and maintaining voltages at the load buses.

5.4 Power flow simulation

Initially the power flow was run with no added distributed generation to check the load bus voltages. The voltages for bus 1 and 2 were 21.318 kV and 21.3774 kV respectively

However the voltage at bus 3 was considerably lower at 20.24 kV and close to the voltage limit. Such a low voltage may lead to problems as this case considers a minimum load. In the case of a peak load the bus voltage will decrease and may well be outside the limits. If more power is needed at the load then a greater current will have to flow which will lead to an increased voltage drop according to the distribution voltage drop approximation

$$V_s - V_r = IR + IX \quad (1)$$

Where $V_s - V_r$ is the voltage drop or the change in voltage and IR and IX are the reactance and resistance of the lines (Brice 1982).

To determine the voltage dependencies of the buses, power was added to each possible DG location in turn. Power was first added incrementally at Bus 1 and the voltages at Bus 1, Bus 2 and Bus 3 were recorded. The system was returned to its original state with no additional DG, and power was then added to Bus 2 while the voltages at Bus 1, Bus 2 and Bus 3 were again recorded. Finally the injected power was once again zeroed at Bus 2 and active power was slowly added to Bus 3, and the voltages at all the buses recorded. This process shows how generation at each bus affects voltage at all the other buses.

The resulting power and voltage relationships for each bus were tabulated and graphed as outlined below.

5.4.1 Power injected into Bus 1

Power injected into Bus 1 (MW)	Bus Voltages (kV)		
	1	2	3
0	21.318	21.3774	20.24
2	21.538	21.417	20.2818
4	21.758	21.4434	20.3082
6	21.978	21.4544	20.3236
8	22.154	21.4566	20.3236
10	22.33	21.4456	20.3126

Table 6 Effect of on Voltages of Power injected at Bus 1

Table 6 above shows the results when increasing levels of distributed generation are added at 2 MW a time to bus 1, and the effect this has on the voltages at the buses in the network. Bus 1 is most affected with the voltage level increasing steadily with increasing generation. For 10 MW of injected power, Bus 1 has a voltage increase of 1.012kV while 2 and 3 are less affected and only have voltage rises of 0.068kV and 0.072kV respectively. This is to be expected as buses 2 and 3 are on a different radial section of the network from bus 1.

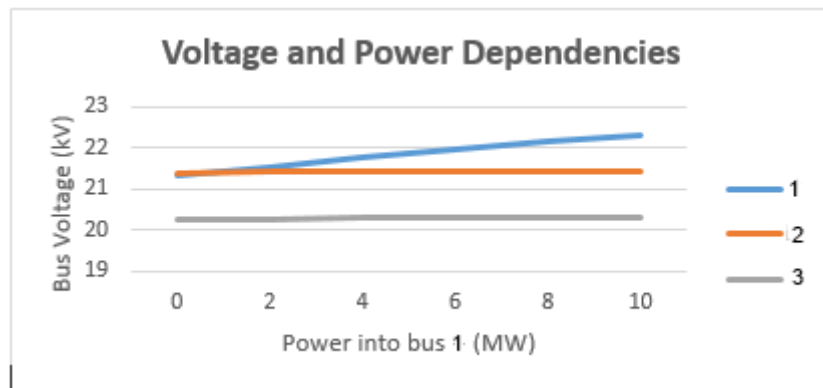


Figure 3 Voltage dependencies of Power injection at bus 1

Figure 3 shows more clearly the impact of the added generation at bus 1 on the bus voltages. The added generation has very little impact on buses 2 and 3, with their voltages rising at approximately the same rate. As the power is incrementally added to bus 1 the voltage at bus 1 rises much faster than in comparison to the other bus voltages.

5.4.2 Power injected into bus 2

Power injected into bus 2 (MW)	Bus Voltages (kV)		
	1	2	3
0	21.318	21.3774	20.24
2	21.34	21.516	20.3852
4	21.384	21.6392	20.5172
6	21.384	21.7514	20.636
8	21.406	21.8526	20.7416
10	21.384	21.9406	20.8362

Table 7 Effect on Voltages of Power injected at Bus 2

Table 7 above shows the results of the voltages at the three buses when increasing levels of power are added to bus 2. Bus 2 has the highest initial voltage due to its location, only 1km from the

transmission station. Bus 1 has a slightly lower initial voltage as it is located 2kms from the transmission station. Bus 2 and 3 have an almost equal increase in voltages over the range of added power increase. Bus 1 is only slightly affected by the generation changes at bus 2, the voltage at bus 1 only increases by 0.066kV, which is to be expected as bus 1 is on a different radial section of the network.

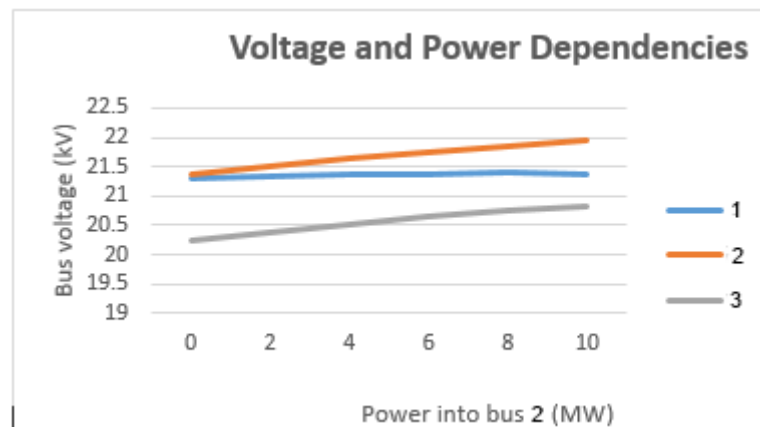


Figure 4 Voltage dependencies of Power injection at bus 2

Figure 4 above shows that bus 2 and 3 voltages are increasing at the same rate. It can also be seen that there is very little change in the voltage levels at bus 1.

5.4.3 Power injected into bus 3

Power injected into bus 3 (MW)	Bus Voltages (kV)		
	1	2	3
0	21.318	21.377	20.240
2	21.340	21.516	22.363
4	21.362	21.600	24.114
6	21.340	21.650	25.637
8	21.296	21.674	26.983
10	21.230	21.679	28.211

Table 8 Effect on Voltages of Power injected at Bus 3

Table 8 above shows the results of the voltages at the three buses when increasing levels of power are added to bus 3. Bus 3 is joined to bus 2 by a 20 km distribution line which is why the initial

voltage is so low. The voltage at bus 1 increases slightly then starts to decrease. The total voltage increase at bus 3 is 7.90 kV which is due largely to the distance of bus 3 from the transmission station.

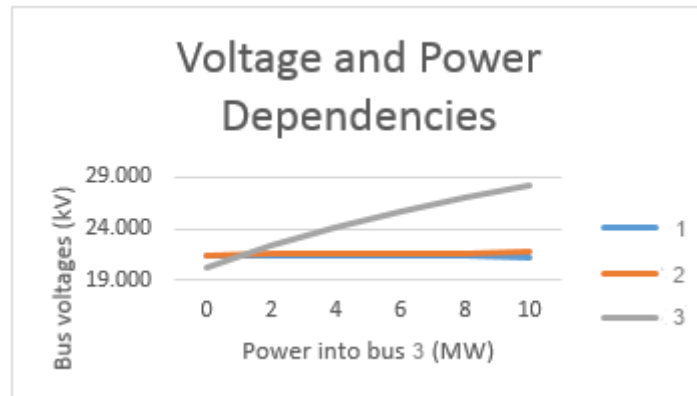


Figure 5 Voltage dependencies of Power injection at bus 3

Figure 5 above shows the bus voltages when power is injected into bus 3. The voltage at bus 3 is most affected by the added power, there is not as great an effect on bus 2 even though they are in the same radial system due largely to the 20 km long distribution line that joins the buses and bus 2's close proximity to the transmission station. When the added power at bus 3 is above 4MW the voltage at the bus then exceeds the $\pm 10\%$ voltage variation limit required by Western Power (Western Power 2008).

5.4.4 Voltage Dependencies

The resulting power and voltage relationships for each bus were graphed and show that the relationship between power and voltage, although not linear, can be approximated by a linear relationship. As power is added to each bus the voltage rises.

When the distributed generation is less than or equal to the loads then the active power produced by the generator flows into the loads and only reactive power is required from the transmission system to supply the loads. When the amount of distributed generation exceeds the load demand then active power enters the transmission system and only reactive power flows from the transmission system to the load. In this case, the distributed generation is injecting power into the grid, which is known as reverse power flow.

It can be seen from the Power Injection/ Voltage Dependency graphs that the greater the line impedance (such as in the 20 km line from bus 2 to 3) and power flow (with increasing levels of generation), then the greater the voltage rise (Keane and O'Malley 2005). The active power flow on distribution networks has a significant effect on the voltage levels due to the high resistive levels of distribution lines compared to transmission lines (Keane and O'Malley 2005).

Power into bus (kV/MW)	Voltage at			Intercept (kV)
	1	2	3	
1	0.099	0.004	0.004	21.318
2	0.006	0.053	0.056	21.377
3	0.011	0.030	0.797	20.240

Table 9 Voltage interdependency

The slopes of the graphs were used to make a voltage interdependency table (Table 9 above) from which the relationship between power and voltage at each Bus can be seen. This table highlights the interdependencies between the power injected at each bus and the voltages at the buses. The values shown in Table 9 above were used in determining the voltage/power relationships for each bus.

The bus voltage power interdependencies were affected by two main factors (Keane and O'Malley 2005)

- The distance from the bus to the transmission station
- The distance between the buses and whether or not the buses were on the same radial feeder.

5.5 Linear equation and constraints

In the equations and constraints below, subscripts 1, 2 and 3 are used to refer to conditions at buses 1, 2 and 3 respectively.

5.5.1 Objective function

The voltage interdependencies and voltage limits at each bus were then used to create voltage limiting equations for the optimisation constrained optimisation problem.

The objective function is to maximise the distributed generation for this power network.

$$J = P_{DG1} + P_{DG2} + P_{DG3} \quad (2)$$

where P_{DGi} is the distributed generation capacity at bus i.

This objective function must be maximised with constraints to limit the generator size and to control voltage rise.

5.5.2 Constraints

5.5.2.1 Voltage levels

There are two main constraints in terms of maximising the DG in order to improve the voltage profile.

Western Power states that voltages must stay within $\pm 10\%$ of the nominal value. In this case voltage at the bus must stay within 5% of the desired voltage in order to show a high accuracy.

$$V_{\min i} < V_i < V_{\max i} \quad (3)$$

Where V_i is the voltage at each bus.

For the 22kV system considered here the minimum voltage is 20.9 kV and the maximum is 23.1 kV, therefore

$$20.9 \times 10^3 < V_i < 23.1 \times 10^3 \quad (4)$$

5.5.2.2 Generation limits

The generation capacity at each bus must be between a minimum and maximum value

$$P_{min DG i} < P_{DG i} < P_{max DG i} \quad (5)$$

$P_{DG i}$ is the generation capacity at each bus. Limits were placed on the generation capacity as it would be unrealistic not to limit the amount of generation.

For the generation capacity at bus 1 the limits were as shown below (MW)

$$0 < P_{DG 1} < 8 \quad (6)$$

For the generation capacity at bus 2 the limits were as shown below (MW)

$$0 < P_{DG 2} < 10 \quad (7)$$

For the generation capacity at bus 3 the limits were as shown below (MW)

$$0 < P_{DG 3} < 5 \quad (8)$$

These limits were chosen arbitrarily but within a realistic range.

5.5.3 Voltage and Power characteristics

The voltage and power characteristics for each bus can be written as

$$V_i = \mu_i P_{EG i} + \beta_i + \sum_{j=1}^N \mu_{ji} P_{EG j} \text{ (Keane and O'Malley 2005)} \quad (9)$$

where μ_i is the dependency on the voltage level at bus i , V_i , on power added to that same bus.

β_i is the bus voltage with no added generation.``````

μ_{ji} is the dependency on the voltage level at bus i of the power injected at bus j .

As described above (Section 5.4.4) the voltage/power relationships for the busses were determined from simulating the injection of power into each bus and noting the resulting voltages. Table 9 Voltage interdependency provides the relationships outlined below.

The voltage/power relationship for bus 1

$$0.099 P_{DG 1} + 21.318 + 0.006 P_{DG 2} + 0.011 P_{DG 3} \leq V_{1,max} \quad (10)$$

The voltage/power relationship for bus 2

$$0.053 P_{DG 2} + 21.377 + 0.004 P_{DG 1} + 0.030 P_{DG 3} \leq V_{2,max} \quad (11)$$

The voltage/power relationship for bus 3

$$0.797 P_{DG\ 3} + 20.24 + 0.004 P_{DG\ 1} + 0.056 P_{DG\ 2} \leq V_{3,max} \quad (12)$$

5.6 Optimisation and Results

The linear equations and constraints were used in a linear optimisation solver in Matlab (Mathworks n.d.) to find the optimum solution and to confirm the accuracy of the model.

The Matlab code is provided at Appendix A.

5.7 Results for Part A of the Model

Generator	Capacity (MW)
1	8
2	10
3	2.845

Table 10 Generated Power – Part 1

Table 10 above shows the optimum location and capacity of DG taking into account voltage stability.

The generation size and location was largely dependent on each buses proximity to the grid.

The optimum power for generators at bus 1 and 2 was limited by the generator bounds that were chosen. As buses 1 and 2 are close to the grid and therefore have a small line resistance, the superposition theorem can be used to understand that the voltage at these two buses was not strongly influenced by the addition of power into the grid. The optimum power at bus 3 was affected by its much greater distance to the grid. As the grid had less influence on the voltage/power relationship at bus 3 so the voltage in this case was strongly influenced by the addition of distributed power.

Generator	Capacity (MW)	Voltage (kV)	Variance from nominal voltage
1	8.00	21.9516	-0.0484
2	10.00	21.8592	-0.1408
3	2.845	23.1044	1.1044

Table 11 Generated power and associated voltages – Part A

When the calculated values for generator output were entered into Power World with minimum load the bus voltages were all maintained within 5% of the nominal value as desired as shown in Table 11 above.

6 Model Part B – line losses

Line losses are an important factor that contribute to the way a distribution system is planned and implemented (Keane and O'Malley 2006). For DG, the objective should be to place and operate the DG in such a way as to maximise DG output in addition to minimising the line losses in a system (Slootweg, et al. 2005)

Keane and O'Mally suggest a way to maximise the generation and reduce the losses in the system using a linear piece wise approximation to model the loss characteristics of a system (Keane and O'Malley 2006). This thesis uses another method to approximate the loss relationship with injected power.

Part B of the model builds on the results from Part A and performs further power flow simulations to develop a non-linear model of power and line loss relationships and then uses both the linear power/ voltage (Part A) and non-linear power / line loss (Part B) relationships in a simplex algorithm to determine the optimum solution for generator location and capacity with minimum power loss while maintaining voltage stability.

6.1 Simplex programming

The objective function is quadratic rather than linear, the objective function was optimised in Scilab (Scilab Enterprises 2014) using the `fminsearch` non-linear optimisation solver which uses the Nelder-Mead simplex search algorithm.

Nelder-Mead is a non-linear optimisation method which uses a changing set of simplices (a simplex is an n dimensional polytope with $n+1$ vertices) in an n dimensional solution space. The method iteratively determines results at the vertices of the current simplex and replaces the “worst” vertex in one of four operations: *reflection*, *expansion*, *contraction*, and *multiple contraction* (Brunet 2010).

Margaret Wright (Wright 2012) describes how the Nelder Mead method is an anomalous singularity in the modern world of search methods, having been subject to improvements since its invention by two statisticians at the British National Vegetable Research Station in the mid-1960s but known to fail in at certain cases. She quotes John Nelder from an interview in 2000:

“There are occasions where it has been spectacularly good. Mathematicians hate it because you can't prove convergence; engineers seem to love it because it often works. (Wright 2012)”

6.2 System model

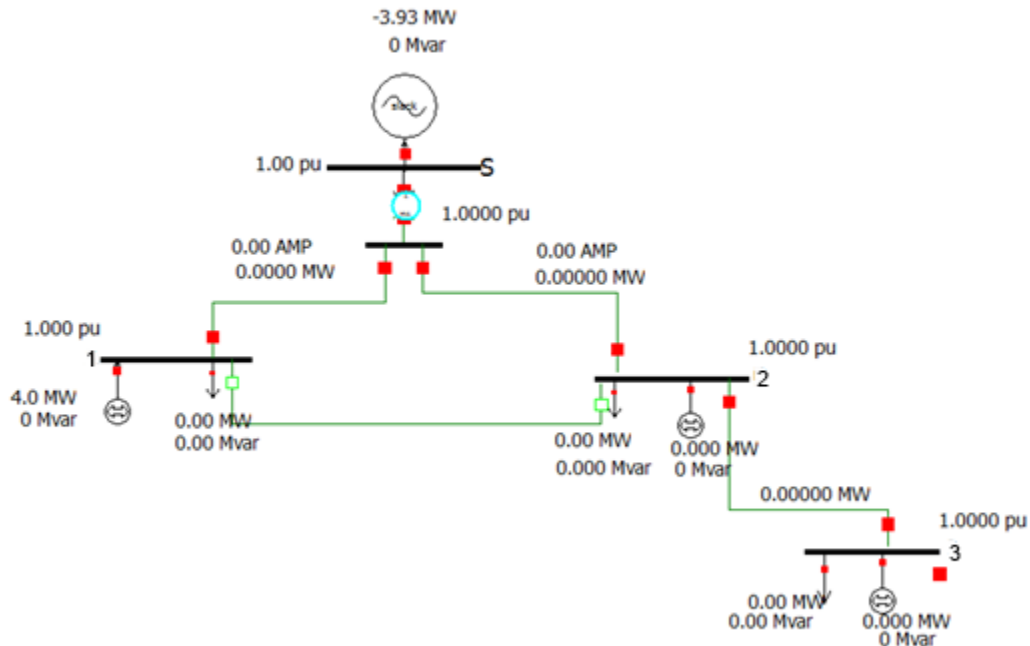


Figure 6 Line diagram with loads at each bus set to zero

Part B uses the same network layout as in Part A, but this time with the loads at each bus set to zero for both active and reactive power. For clarity the system is shown again at Figure 6 above.

An accurate calculation of losses would require a power flow analysis at every trial point during the search for the maximum value of J , which is the objective function. As an alternative approximation, the loss function is often modelled as a quadratic function of the generated powers (Glover 2012, 675-676). The simplest model ignores the interaction between generators, and is written as

$$P_{losses} = \sum_i a_i P_{DGi}^2 \quad (13)$$

The coefficients $\{a_i; i = 1..n\}$ can be estimated by carrying out a sequence of load flow analyses. Initially the power output of each generator (except the grid) and loads are set to zero, then the output at each generator in turn is varied and the resultant losses around the network are recorded. From the plots of losses versus P_{DGi} , a best fit to a quadratic equation is obtained, from which an estimate for a_i is calculated.

6.2.1 Power injected into Bus 1

Power injection at 1 (MW)	Line Losses (MW)
0	0
0.5	0.0011

1	0.0043
1.5	0.0097
2	0.0171
2.5	0.0267
3	0.0382
3.5	0.0518
4	0.0674
5	0.1044
6	0.1492
7	0.2016

Table 12 Power and line losses for power injections at bus 1

Table 12 above shows the resulting line losses for varying injected power levels at bus 1. The only losses in the system were associated with the line that connects the grid to bus 1, as would be expected with no load or generation in the rest of the system.

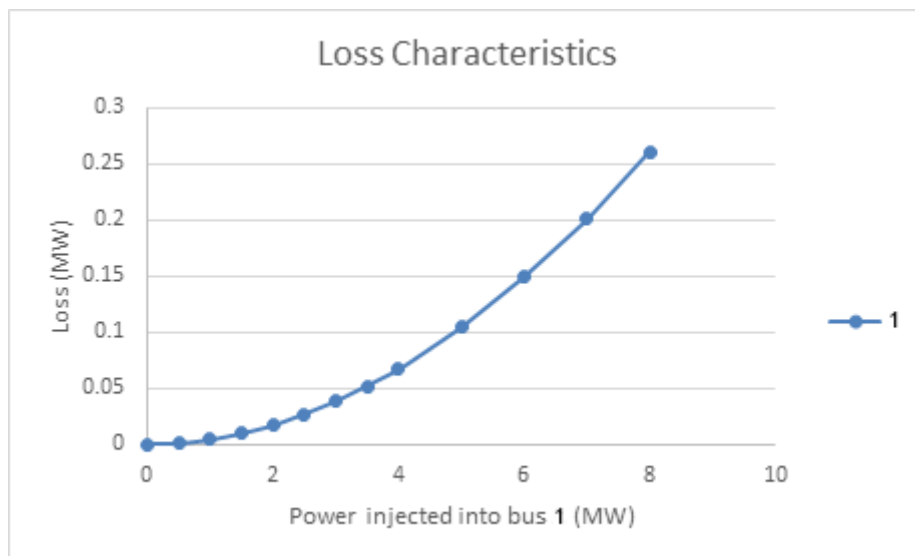


Figure 7 Graph of power and line losses for power injections at bus 1

The plot of loss versus power injected into bus 1 can be seen at Figure 7 above showing a quadratic form as expected. Although all the losses in the system caused by varying the power just at bus 1 were determined, losses occurred only on the line from the grid to bus 1. This is to be expected as bus 1 is on a different radial section to bus 2 and 3 and therefore any power injected into bus 1 had little effect on the other line losses. Using the shape of the line the value for the coefficient of power loss, a_1 , was found to be 0.004628.

6.2.2 Power injected into Bus 2

Power injection at 2 (MW)	Line Losses (MW)
0	0

1	0.00217
2	0.00864
3	0.01934
4	0.03424
5	0.05327
6	0.07642
7	0.10364

Table 13 Power and line losses for power injections at bus 2

Table 13 above shows the resulting line losses for varying injected power levels at bus 2. The only losses in the system were associated with the line that connects the grid to bus 2, as would be expected with no load or generation in the rest of the system.

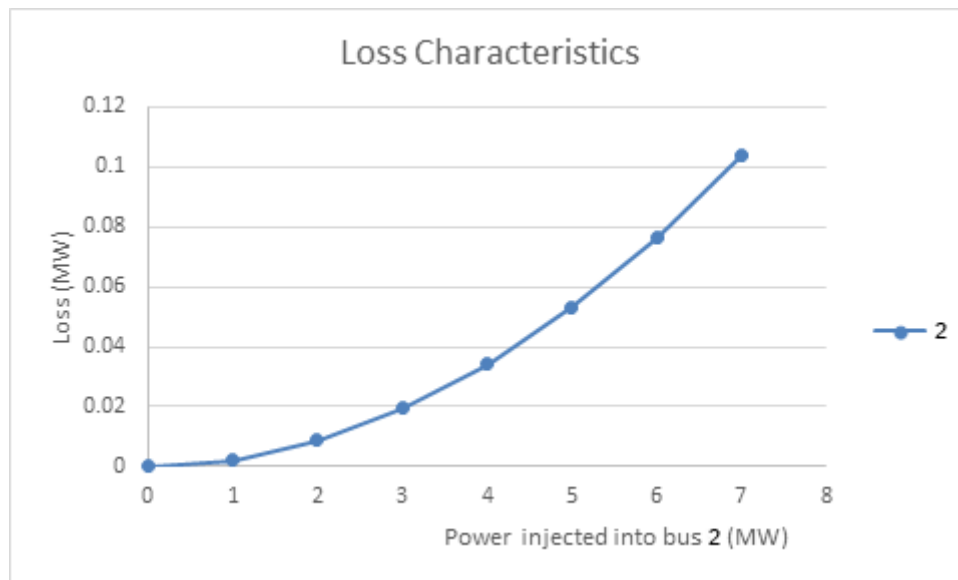


Figure 8 Graph of power and line losses for power injections at bus 2

The plot of loss versus power injected into bus 2 can be seen at Figure 8 above showing a quadratic form as expected. Although all the losses in the system caused by varying the power just at bus 1 were determined, losses occurred only on the line from the grid to bus 1. This is to be expected as bus 1 is on a different radial section to bus 2 and there was no load at bus 3 so no reason for power to flow in that direction. Using the shape of the line the value for the coefficient of power loss, a_2 , was found to be 0.002819.

6.2.3 Power injected into Bus 3

Power injection at 3 (MW)	Line Loss Line 3 (MW)	Line Loss Line 2 (MW)	Total Losses (MW)
0	0	0	0
1	0.04003	0.002	0.04203

2	0.14823	0.00741	0.15564
3	0.31242	0.01562	0.32804
4	0.52158	0.02608	0.54766
5	0.77081	0.03854	0.80935

Table 14 Power and line losses for power injections at bus 3

Table 14 above shows the resulting line losses for varying injected power levels at bus 3. The losses in this case occur both in the line from bus 3 to bus 2 and in the line from bus 2 to the grid. Even though the generator at bus 2 is not generating any power, the power from the generator at bus 3 flows towards the grid through the line from bus 2 to the grid, thus producing line losses in this network segment.

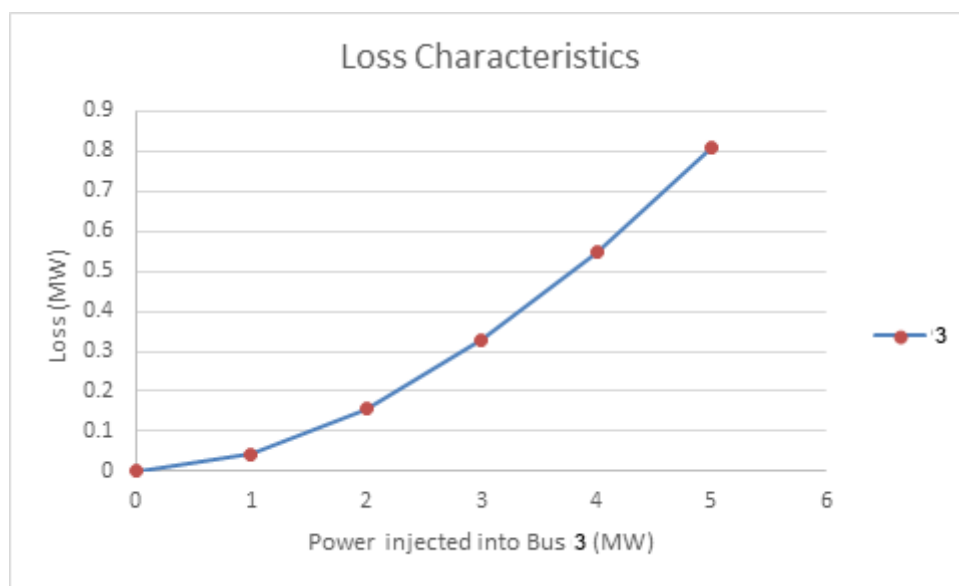


Figure 9 Graph of power and line losses for power injections at bus 3

The plot of loss versus power injected into bus 3 can be seen at Figure 9 above showing a quadratic form as expected above shows the system losses versus power added to bus 3. Again the shape of the curve is quadratic. Using the shape of the line the value for the coefficient of power loss, a_3 , was found to be 0.032374.

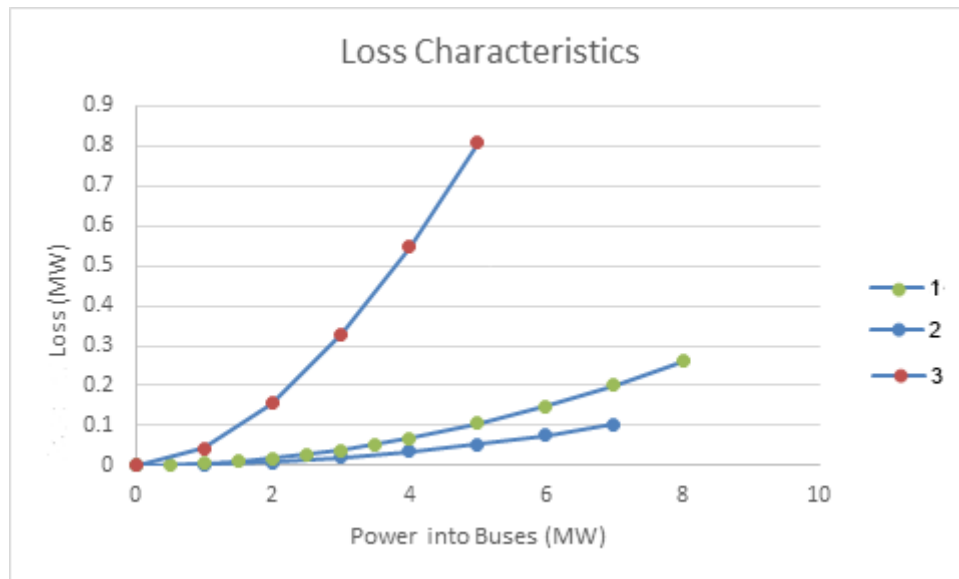


Figure 10 Overall loss characteristics

Figure 10 above shows the resulting line losses associated with varying power injection into each bus. As there are no loads in the system all the current generated by the DG sources flows towards the grid. Line losses are dependent on current flowing through the line and resistance of the line and vary according to I^2R which explains the quadratic shape of the graphs. As clearly seen in the graph the losses associated with injecting power into bus 3 increase at a greater rate than the losses associated with injecting power into either of the other two buses. Resistance is proportional to length of the line, as the length of line 2-3 is 20km it has a higher resistance than the line from 1 to the grid (2 km) or from 2 to the grid (1km) and this higher resistance results in much greater line losses. Similarly the losses associated with bus 1 (2 km from the grid) are slighter greater than those for bus 2 (1 km from the grid).

Bus	Line Loss coefficient
1	0.004628
2	0.002819
3	0.032374

Table 15 line loss coefficients

The coefficients determined from the simulations are shown at Table 15 above.

6.3 Quadratic equation and constraints

6.3.1 Objective function

As in Part A, in the equations and constraints below, subscripts 1, 2 and 3 are used to refer to conditions at buses 1, 2 and 3 respectively.

As from Part A the proposed objective function is (Keane and O'Malley 2005)

$$J = \sum_i P_{DGi} \quad (2)$$

The optimal allocation task is to maximize this function subject to a number of constraints, including the constraint of keeping the voltages at all buses within prescribed limits and the injected power from the generators within certain bounds.

This function assumed that all the power that was generated within the network would either be absorbed by the loads or fed into the grid.

With the inclusion of transmission losses the function that is to be maximised is

$$J = \sum_i P_{DG_i} - P_{losses} \quad (14)$$

The loss functions can be modelled as simple quadratic functions of the generated powers. The simplest model is written as

$$P_{losses} = \sum_i a_i P_{DG_i}^2 \quad (13)$$

Since

$$\sum_i P_{DG_i} - \sum_i a_i P_{DG_i}^2 = \sum_i (P_{DG_i} - a_i P_{DG_i}^2) \quad (15)$$

For this model, the objective function to be maximised thus becomes

$$J = \sum_i (P_{DG_i} - a_i P_{DG_i}^2) \quad (16)$$

As described above the coefficients $a_1...a_3$ were identified by carrying out a sequence of load flows and plotting losses versus (P_{DG_i}) then determining a best fit of the quadratic to find each loss coefficient a_i

The objective function for this system is then

$$J = (P_{DG1} - 0.004628) + (P_{DG2} - 0.002819P_{DG2}^2) + (P_{DG3} - 0.003237P_{DG3}^2) \quad (17)$$

This function then maximises the generation at each bus while minimising the losses associated with the generation.

This objective function must be maximised with constraints to limit the generator capacity and to control voltage rise.

6.3.2 Constraints

6.3.2.1 Voltage levels

The bus voltage limits are as in Part A of the model, refer to section 5 above for more details. The limits for each bus can be seen below (kV)

$$20.9 < V_i < 23.1 \quad (4)$$

These limits are the same for all three voltages, so will be used for V_1 , V_2 and V_3 .

6.3.2.2 Generation limits

The generation capacity limits at each bus are as in Part A, refer to section 5 above for more information.

For the generation capacity at bus 1 the limits were as shown below (MW)

$$0 < P_{DG\ 1} < 8 \quad (6)$$

For the generation capacity at bus 2 the limits were as shown below (MW)

$$0 < P_{DG\ 2} < 10 \quad (7)$$

For the generation capacity at bus 3 the limits were as shown below (MW)

$$0 < P_{DG\ 3} < 5 \quad (8)$$

These limits were chosen arbitrarily but within a realistic range.

In a realistic scenarios these ranges could be chosen based on numerous things, such as cost and the size of the load the generators were to supply.

6.3.3 Inequalities

The voltage inequalities as found in Part A (section 5 above) are used again to specify the relationship between power and voltage at each bus.

The voltage/power relationship for bus 1

$$0.099 P_{DG\ 1} + 21.318 + 0.006 P_{DG\ 2} + 0.011 P_{DG\ 3} \leq V_{1,max} \quad (10)$$

The voltage/power relationship for bus 2

$$0.053 P_{DG\ 2} + 21.377 + 0.004 P_{DG\ 1} + 0.030 P_{DG\ 3} \leq V_{2,max} \quad (11)$$

The voltage/power relationship for bus 3

$$0.797 P_{DG\ 3} + 20.24 + 0.004 P_{DG\ 1} + 0.056 P_{DG\ 2} \leq V_{3,max} \quad (12)$$

6.3.4 Optimisation

The Nelder-Mead method stops further iterations when

$$\sqrt{\frac{1}{n+1} \sum_{i=0}^n \left(f(\mathbf{x}_i) - \overline{f(\mathbf{x}_i)} \right)^2} \leq \varepsilon \quad (18)$$

where ε is a small fixed constant (eg $\varepsilon = 10^{-6}$) and n is some chosen averaging window.

The Scilab code is provided at Appendix B.

6.4 Results for Model Part B

Generator	Capacity (MW)
1	8.00
2	10.00
3	2.834

Table 16 Generated Power – Part B

Table 16 above shows the location and capacity of DG the optimum location and capacity of DG taking into account voltage stability and minimising line losses.

The objective function is to maximise the distributed generation taking account of line losses, which can be thought of as a reducing a portion of the total generation. As in Part A the optimal power generation was still influenced by the bus distance to the grid in regards to voltage control, but the results from Part B shows that distance from the grid also affects line losses with a bus close to the grid (e.g. bus 1) experiencing lower line losses than another bus which lies a considerable distance from the grid (bus 3).

The power output for both bus 1 and 2 was once again determined by the generator limits as the voltages are not greatly affected by injected power, and the line losses are limited as both buses are close to the grid. When the generator constraints for bus 1 and 2 were relaxed the capacity for them both was above 50MW.

As bus 3 is the furthest from the grid the injected power has a greater effect on the bus voltage resulting in a lower power output. Line losses are also greatest for bus 3 and the generated output was reduced from the results derived in Part A where line losses were not considered, although there is only a very small difference.

Generator	Capacity (MW)	Voltage (kV)
1	8.00	21.9516
2	10.00	21.8592
3	2.834	23.1044

Table 17 Generated power and associated voltages – Part B

When these calculated values for generator size were once again entered into Power World, and the minimum load set, the bus voltages were all maintained within 5% of the nominal value as desired, as can be seen in Table 17 above.

The loss functions do not take into account the relationships between the generators, in particular relationship between bus 2 and 3 which are on the same radial section. An attempt was made to model the relationship between the power injected at bus 2 and the power added at bus 3 but this did not appear to have any effect on the system and these results are not included.

Model	Bus Capacity (MW)			Total Loss (MW)
	1	2	3	
Part A	8	10	2.845	0.566
Part B	8	10	2.834	0.540

Table 18 Part A and Part B results compared

Table 18 above shows a comparison between the system losses when the generator output is determined excluding and including line losses. It can be seen that the generator output in Part B reduces overall system losses, although only slightly. Output at buses 1 and 2 is not affected by line losses because of the proximity of the buses to the grid.

7 Future work

The model presented in this thesis achieves the required results but uses a simplistic approach. The inclusion of other parameters, most importantly thermal limits, transformer limits and short circuit ratings would improve its usefulness and accuracy. It is also important to model the generators as renewable as their allocation would be dependent on their generation profile.

Thermal limits on overhead lines are established to prevent the lines overheating and stretching, and affect the maximum line current (Larruskain, et al. 2002). The addition of a DG source in a network will generally increase the current in the system (Lai and Chan 2007). DG placement and sizing needs to take account of the thermal limits of the lines.

Transformers operate within a certain power range which should be considered in the model as exceeding the power limit can significantly reduce the life of the transformer (Keane and O'Malley 2005).

The equipment in distribution systems has a maximum short circuit rating, which is the highest short circuit current that the system can accept and still operate properly (Glover, Sarma and Overbye 2008). The addition of DG sources can increase the fault level in a system (Lai and Chan 2007). This model should incorporate a constraint that the short circuit ratings of the system cannot be exceeded.

The generation sources should be modelled as renewable energy sources with a varying power profile to improve the accuracy of the model.

8 Conclusion

The model for the optimisation of distributed generation developed through this project satisfies the objectives, which were to provide a practical method of optimising distributed generation taking account of voltage stability requirements and minimising line losses.

It is important to plan the placement of DG sources as they can not only negatively impact the grid, but could possibly be used to improve the systems efficiency and improve the voltage profile.

This model reduces the time taken to resolve a power flow optimisation problem by:

- performing a minimal number of power flow simulations to develop:
 - a linear model of power loss and voltage,
 - a non-linear model of power and line loss relationships
- using a simplex algorithm to the model to determine the optimum solution for generator location and output with minimum power loss while maintaining voltage stability.

The final model is able to determine an optimum location and output of the distributed generation units that satisfy the required objectives and constraints.

Table 16 shows the calculated optimum values for maximum distributed generation while reducing line losses and maintaining voltages within limits. These generator values maintain the voltage and reduce the losses of the system in comparison with random placement.

Appendix A: Matlab code

Appendix A shows the Matlab code used for part A, the linear programming used to find the generator capacity and placements. This code uses upper bounds and lower bounds for the bus voltages and generator sizes. There are also three inequality statements which use the voltage/power linear approximations.

Matlab code

```
lb = zeros(6,1);
lb(5) = 19.8;
lb(4) = 19.8;
lb(6) = 19.8;
ub = Inf(4,1);
ub(1) = 10;
ub(2) = 10;
ub(3) = 5;
ub(4) = 22;
ub(5) = 22;
ub(6) = 22;
A = zeros(3,6); b = zeros(3,1);
A(1,1) = .099; A(1,2) = .006; A(1,3) = .011; A(1,4) = -1; b(1) = -21.318;
A(2,1) = .004; A(2,2) = .053; A(2,3) = .030; A(2,5) = -1; b(2) = -21.3774;
A(3,1) = .004; A(3,2) = .056; A(3,3) = .797; A(3,6) = -1; b(3) = -20.24;
Aeq = zeros(2,6); beq = zeros(2,1);
f = zeros(6,1);
f(1) = -1;
f(2) = -1;
f(3) = -1;
[x fval] = linprog(f,A,b,Aeq,beq,lb,ub);
```

Appendix B: Scilab code

Appendix B shows the Scilab code used for part A, the simplex algorithm used to find the generator capacity and placements in order to reduce line losses.

Scilab code

```
function [f, index]=carrot(x, index)

// loss equations
PL1 = 4.0843*10^-3*x(1)^2;
PL2 = 2.115*10^-3*x(2)^2;
PL3 = (3.236*10^-2)*x(3)^2;

V1 = .099*x(1)+21.318+.006*x(2)+.011*x(3);
V2 = .004*x(1)+21.3774+.053*x(2)+.03*x(3);
V3 = .004*x(1)+20.24+.056*x(2)+.797*x(3);

// the function is minimising anything positive that is why the x's have - in front, so they can be maximised.
f = -x(1)-x(2)-x(3)+PL1+PL2+PL3;
if x(1)> 8 then f = 10^7
end
if (x(1)< 0) f = 10^6
end
if x(2)> 10 then f = 10^8
end
if (x(2)< 0) f = 10^6
end
if x(3)>5 then f = 10^8
end
if (V1 > 23.1) then f = 10^7
end
if (V1 < 20.9) then f = 10^7
end
if (V2 < 20.9) then f = 10^7
end
if (V2 > 23.1) then f = 10^7
end
if (V3 < 20.9) then f = 10^7
end
if (V3 > 23.1) then f = 10^7
end

endfunction
[x, fval, exitflag, output] = fminsearch (carrot, [8 8 2])
mprintf('%6.3f %6.3f %6.3f', x)
```

9 References

- Brice, C. W. 1982. "COMPARISON OF APPROXIMATE AND EXACT VOLTAGE DROP CALCULATIONS FOR DISTRIBUTION LINES." *IEEE Transactions on Power Apparatus and Systems* 4428-4436.
- Brunet, Florent. 2010. *Basics on Continuous Optimization* . November. Accessed July 05, 2014. <http://www.brnt.eu/phd/node10.html#SECTION00622200000000000000>.
- Eckles, Steve. 2007. *Reducing Distribution Losses Without Breaking the Bank*. Company Report, New Mexico: El Paso Electric Co.
- Ferguson, Thomas S. n.d. *LINEAR PROGRAMMING A Concise Introduction*. Accessed July 05, 2014. <http://www.math.ucla.edu/~tom/LP.pdf>.
- Global Wind Energy Council. 2013. *Global Status Overview*. November. Accessed August 22, 2014. <http://www.gwec.net/global-figures/wind-energy-global-status/>.
- Glover, Duncan. J, Mulukatla. S Sarma, and Thomas. J. Overbye. 2008. *Power System : Analysis and Design*. Stamford: Cengage Learning.
- Holttinen, Hannele, and Ritva Hirvonen. 2005. "Power System Requirements for Wind Power." In *Wind Power in Power Systems*, by Thomas Ackermann, 143-165. Chichester: John Wiley and Sons.
- ITP. 2010. "ADDRESSING GRID-INTERCONNECTION WITH VARIABLE RENEWABLE ENERGY SOURCES." December. <http://www.itpau.com.au/wp-content/uploads/2012/08/ITP-APEC-Web-Summary.pdf>.
- Jenkins, Nick, Ron Allan, Peter Crossley, Daniel Kirschen, and Goran Strbac. 2010. *Embedded Generation*. London: The Institute of Electrical Engineers .
- Keane, A, and O. O'Malley. 2005. "Optimal Allocation of Embedded Generation on Distribution Networks." *IEEE Transactions on Power Systems* (IEEE Transactions on Power Systems) 1640-1646.
- Keane, Andrew, and Mark O'Malley. 2006. *Impact of Distributed Generation Capacity on Losses* 1-7.
- Lai, Loi Lei, and Fun Tze Chan. 2007. *Distributed Generation* . Chichester: John Wiley & Sons, Ltd.
- Larruskain, D. M, L Zamora, O Abarategui, A Iraolagoitia, M. D Gutiérrez, E Loroño, and F Bodega. 2002. *Power transmission capacity upgrade of overhead lines*. Accessed July 4, 2014. http://www.icrepq.com/icrepq06/296_Larruskain.pdf.
- Lewis, Simon. 2012. *Carnarvon: A case Study of Increasing Levels of PV Penetration in an Isolated Electricity Supply System*. UNSW Centre for Energy and Environmental Markets for the Australian PV Association.
- Mathworks. n.d. *Matlab : The Language of Technical Computing*. Accessed August 23, 2014. <http://www.mathworks.com.au/products/matlab/>.

- Milligan, Michael, Kevin Porter, Edgar DeMeo, Paul Denholm, Hannele Hottinen, Brendan Kirby, Nicholas Miller, et al. 2012. "Preface; Wind Power Myths Debunked." In *Wind Power in Power Systems*, by Thomas Ackermann, 7-15. Chichester: Wiley & sons.
- Northcote-Green, James, and Robert Wilson. 2007. "Voltage Control." In *Control and Automation of Electrical Power Distribution Systems*, by James Northcote-Green and Robert Wilson, 121-135. Sound Park NW: Taylor and Francis.
- Parmar, Jignesh. 2013. *Total Power Losses in Power Distribution and Transmission Lines*. March. Accessed June 28, 2014. <http://electrical-engineering-portal.com/total-losses-in-power-distribution-and-transmission-lines-1>.
- Pisica, Ioana, Petru Postolache, and Marcus M Edvall. 2010. "Optimal planning of distributed generation via non linear optimization and genetic algorithms." In *Handbook of Power Systems (1)*, by Steffan Rebennach, Panos M Pardalous and Mario V.F Pereira, 451-459. London : Springer.
- Power World. 2012. *Power World*. Accessed August 22, 2014. <http://www.powerworld.com/>.
- Santjer, Fritz. 2005. "Power Quality Measurements." In *Wind Power in Power Systems*, by Thomas Ackermann, 102-108. Chichester: John Wiley and Sons.
- Scilab Enterprises . 2014. *Scilab*. Accessed June 15, 2014. <http://www.scilab.org/>.
- Slootweg, J.G, S. W.H Haan, H Polinder, and W. L. Kling. 2005. "Wind Power and Voltage Control." In *Wind Power in Power Systems*, by Thomas Ackermann, 413-432. Chichester: John Wiley and Sons.
- Synergy. 2014. *Kalbarri Wind Farm*. Accessed August 22, 2014. <http://generation.synergy.net.au/generating-electricity/sustainable-portfolio/kalbarri-wind-farm>.
- US Department of Energy. 2012. *NATIONAL ELECTRIC TRANSMISSION CONGESTION STUDIES*. 23 June. <http://energy.gov/oe/services/electricity-policy-coordination-and-implementation/transmission-planning/national>.
- Verve Energy. 2006. *Kalbarri end-of-grid wind farm*. Accessed August 22, 2014. http://services.verveenergy.com.au/mainContent/sustainableEnergy/OurPortfolio/Kalbarri_End-of-Grid_Wind_Farm.html.
- Western Power. 2008. "User Guide for the connection of generators of up 10MW to the Western Power SWIN distribution system." *Western Power Connection Guide*. July. Accessed August 22, 2014. <http://www.westernpower.com.au/documents/User%20guide.pdf>.
- Wright, Margaret H. 2012. "Nelder, Mead, and the Other Simplex Method." *Documenta Mathematica* 271-276.